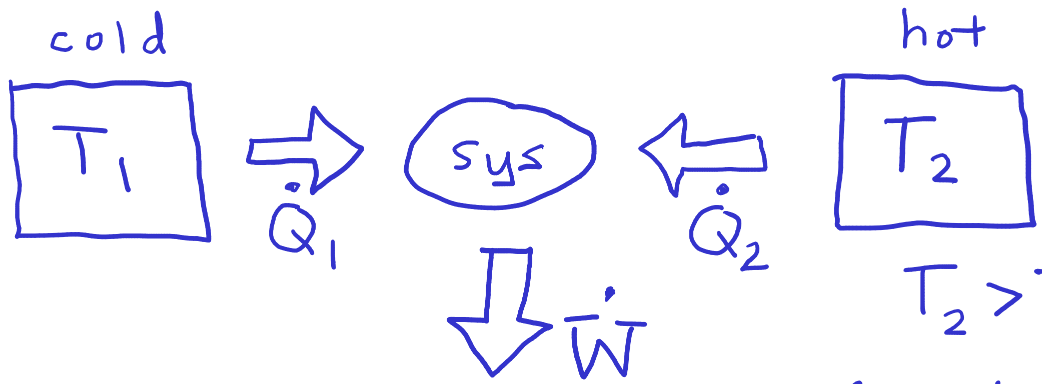
PHYS 414: 3-30-20



$T_2 > T_1$

energy conservation: $\dot{Q}_1 + \dot{Q}_2 = \dot{E} + \dot{W}$

entropy budget: $\dot{S}^i = \dot{S} - \dfrac{\dot{Q}_1}{T_1} - \dfrac{\dot{Q}_2}{T_2} \geq 0$

stat. state: $\dot{S} = 0$, $\dot{E} = 0$

<u>case 1:</u> $\dot{Q}_2 > 0$ (hot bath donates heat)

$$\frac{\dot{W}}{\dot{Q}_2} = \underset{\text{effic.}}{\eta} = 1 - \frac{T_1}{T_2} - \frac{T_1 \dot{S}^i}{\dot{Q}_2}$$

$$\leq 1 - \frac{T_1}{T_2}$$

Carnot inequality

$\Rightarrow \dot{W} \leq \dot{Q}_2$

$\dot{Q}_1 + \dot{Q}_2 = \dot{W}$

waste

$\dot{Q}_1 = \dot{W} - \dot{Q}_2 \leq 0$ (energy dumped into cold bath)

To achieve Carnot efficiency: $\dot{S}^i \to 0$

$$\dot{S}^i = \frac{k_B}{2} \sum_{nmk} J^{(k)}_{nm} \ln \frac{W^{(k)}_{nm}\, p_m}{W^{(k)}_{mn}\, p_n} \geq 0$$

only way to achieve $\dot{S}^i \to 0$
is to have all
currents $J^{(k)}_{nm} \to 0$

but then all dotted quanties $\to 0$

i.e. $\dot{Q}_k = \frac{1}{2} \sum_{nmk} J^{(k)}_{nm} (E_n - E_m + V^{(k)}_{nm})$

$$\to 0$$

$$\dot{\overline{W}} \to 0$$

this is a <u>very</u> boring limit:
approach Carnot effic.
as power output $\dot{\overline{W}} \to 0$

Recall cyclic driving $W_{nm}(t) = W_{nm}(t+\tau)$

analogous formulation of above derivation:

$$\dot{A} \xrightarrow[\text{replace}]{} \Delta A$$

difference over one cycle

$$\dot{\overline{E}} = \dot{S} = 0 \to \Delta \overline{E} = \Delta S = 0, \text{ etc.}$$

analogous Carnot bound:

work over one cycle $\rightarrow$ $$\frac{\Delta W}{\Delta Q_2} \leq 1 - \frac{T_1}{T_2}$$ applies to cyclic engines

achieve Carnot effic when $\Delta S^i = \int_0^\tau \dot{S}^i \, dt \rightarrow 0$

(very slow driving $\Rightarrow$ "quasistatic")

$\tau \rightarrow \infty$ (cycle gets long)

$$\frac{\Delta W}{\tau} \rightarrow 0$$ mean power $\rightarrow 0$

Interesting question:

Max. effic $\Longleftrightarrow$ Zero power

What is effic. at max. power?

Here no known universal bound.

System-specific?

Case 2: $\dot{Q}_1 > 0$ (cold bath gets colder)

refrigerator!

entropy budget w/ $\dot{S} = 0$ $\quad \dot{S}^i = -\frac{\dot{Q}_1}{T_1} - \frac{\dot{Q}_2}{T_2}$

$$\Rightarrow \dot{Q}_2 = -T_2\left(\frac{\dot{Q}_1}{T_1} + \dot{S}^i\right) < 0$$

hot bath gets hotter (machine is also a "heat pump")

$$|\dot{Q}_2| > |\dot{Q}_1| \Rightarrow \boxed{\dot{W} = \dot{Q}_1 + \dot{Q}_2 < 0}$$

law of plugging in the refrigerator (ext. env. has to do work on sys.)

refrig. coeff. $\eta_R = \frac{\dot{Q}_1}{-\dot{W}} = \frac{\text{heat rate out of cold bath}}{\text{power input}}$

$$= \frac{T_1}{T_2 - T_1}\left(1 + \frac{T_2 \dot{S}^i}{\dot{W}}\right)$$

real
refrig.

$\eta_R \gtrsim 4$

$$\eta_R \le \frac{T_1}{T_2 - T_1} \approx 11$$

$T_2 = 298$ K

$T_1 = 273$ K

$\Rightarrow$

heat pump coeff. $\eta_H = \dfrac{-\dot{Q}_2}{-\dot{W}} = \dfrac{\text{heat pump rate}}{\text{power input}}$

$$= 1 + \eta_R$$