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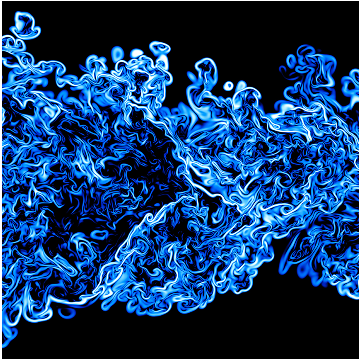
- **Gravity is weak:** A 400 nm droplet descends on average 1 m every 41 hours.

Brownian vs. turbulent diffusivity



Normal ("Brownian") diffusion coefficient is quite small for sub-micron droplets: $D_B < 10^{-10}$ m^2/s, which means < 0.1 mm spread per hour for still air with only thermal fluctuations.

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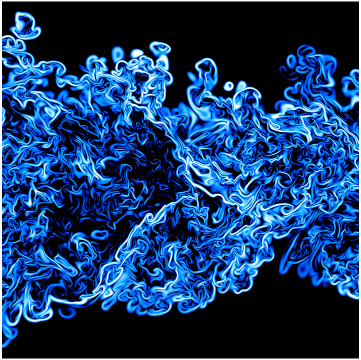

Turbulent eddies in air moving at velocity v_{air} create an effective diffusivity D_T that is much higher. Indoor Chen-Xu phenomenological model:

$$D_T = 0.03874 v_{\text{air}} L \quad (1)$$

where $L \sim 1$ m is the characteristic distance scale to the walls of the room. For typical air conditioning, $v_{\text{air}} = 0.2$ m/s and $D_T \approx 10^{-2}$ m^2/s.

Equation for probability distribution

$$\frac{\partial}{\partial t} p(x,t) = -v_{\text{air}} \frac{\partial}{\partial x} p(x,t) + D \frac{\partial^2}{\partial x^2} p(x,t) \tag{2}$$

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If we scale v_{air} up by a factor of c, then $D \approx D_T$ also increases by a factor of c:

$$\frac{\partial}{\partial t}p(x,t) = -cv_{\text{air}}\frac{\partial}{\partial x}p(x,t) + cD\frac{\partial^2}{\partial x^2}p(x,t) \tag{3}$$

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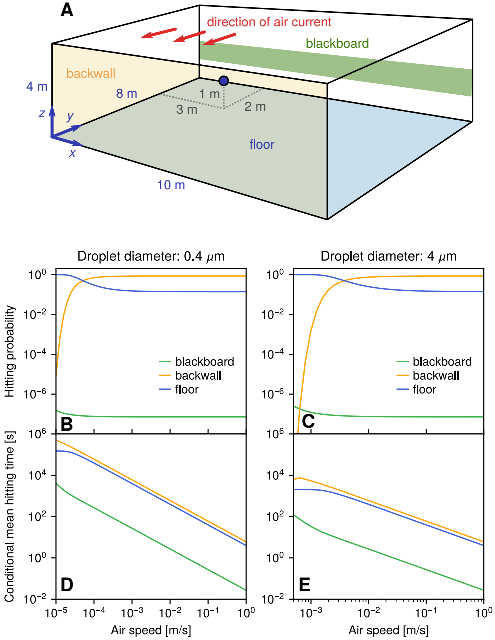
The solution then looks like:

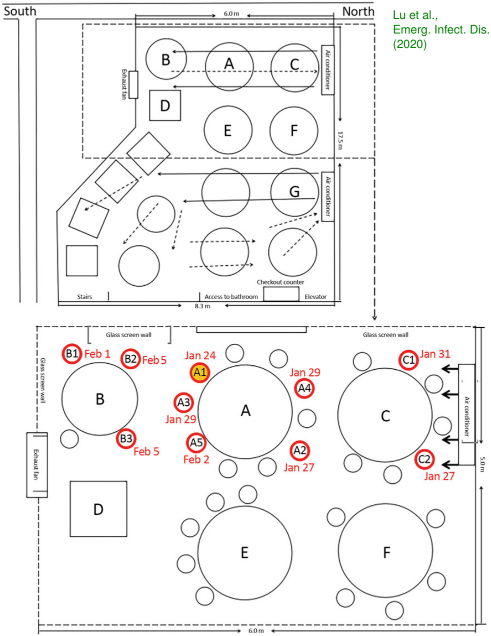
$$p(x,t) = \frac{1}{\sqrt{4\pi cDt}}\exp\left(-\frac{(x-(x_0+cv_{\text{air}}t))^2}{4cDt}\right). \tag{4}$$

Note that it has the same shape and behavior as the original ($c = 1$), except that **time is effectively sped up by a factor of** c. Thus relative probabilities of hitting boundaries remain the same (but the droplets reach the boundaries faster).

Hitting probabilities and mean hitting times



Case study from a restaurant in Guangzhou, China



Lu et al.,
Emerg. Infect. Dis.
(2020)