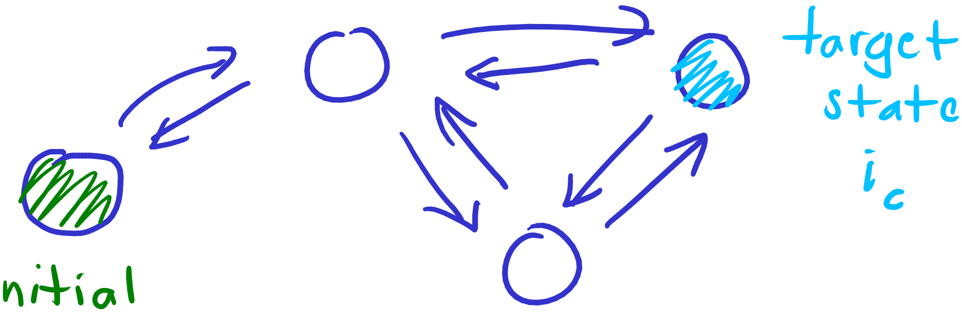
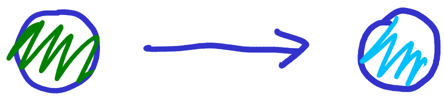
PHYS 320/420 Lecture 8

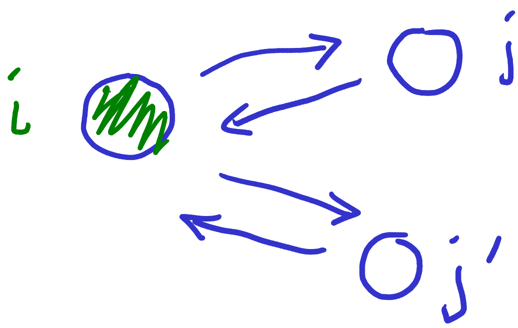


τ_i = avg. time it takes to get from i to i_c for first time



Break problem into three parts:

1) how long on avg to leave initial state



2) which state do you visit after leaving i? (probability)

3) recursive argument $\Rightarrow$ use results of 1) + 2) to build an equation for τ_i

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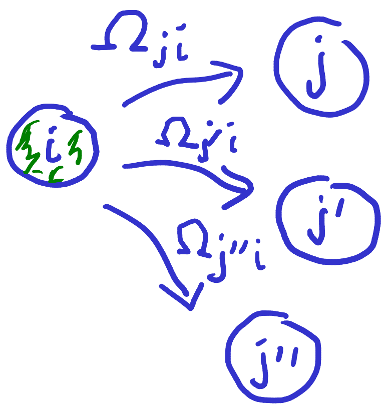
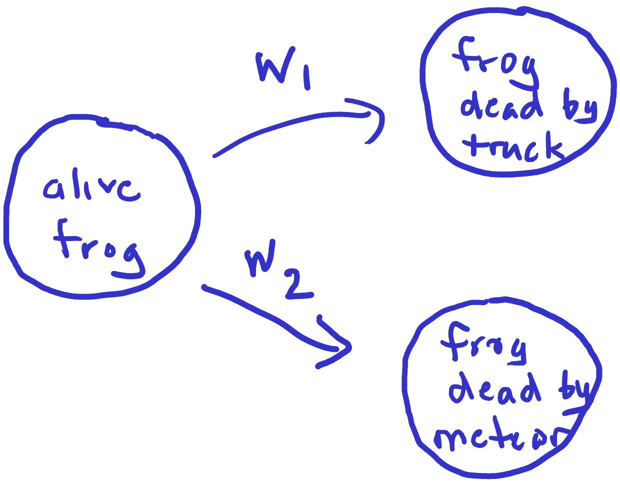
Element #1



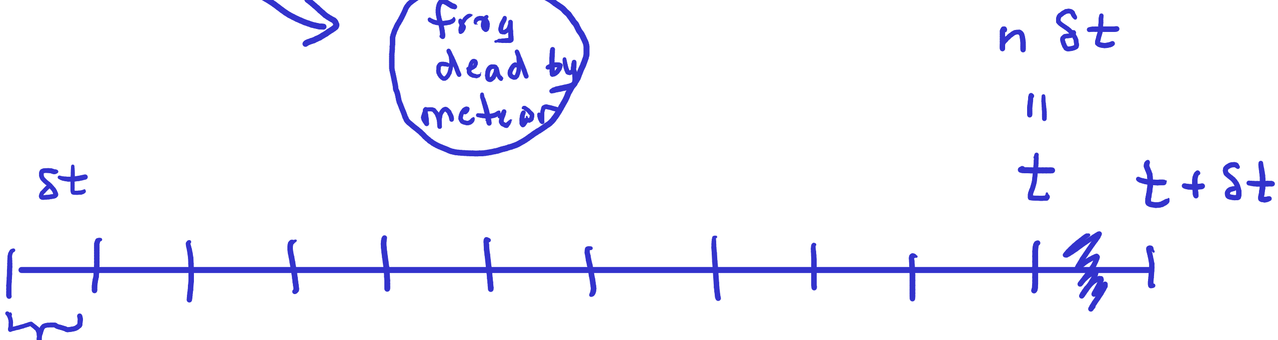
$$\frac{.3}{.3+.1} = 0.75$$

$$\delta t = 1s$$



$$w_1\,\delta t = 0.05$$

$$w_2\,\delta t = 10^{-20}$$



prob. of not leaving $i = 1-(w_1+w_2)\,\delta t$

prob. that you leave state i exactly between time t + $t+\delta t$ $\equiv f_i(t)\,\delta t$

note: $\int_0^\infty dt\, f_i(t) = 1$

$$f_i(t)\,\delta t = \underbrace{\left[1-(w_1+w_2)\,\delta t\right]^n}_{\text{prob. survive } n \text{ time steps}}\ \underbrace{(w_1+w_2)\,\delta t}_{\text{prob. you die in next time step}}$$

in general,

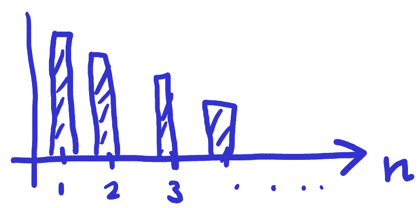
prob. of leaving i in time step δt

$$= \sum_{j\neq i} \Omega_{ji}\,\delta t = |\Omega_{ii}|\,\delta t$$

recall: $\Omega_{ii} = -\sum_{j\neq i} \Omega_{ji}$ (b/c columns sum to zero)

prob. of not leaving i " " :

$$1 - |\Omega_{ii}|\,\delta t$$



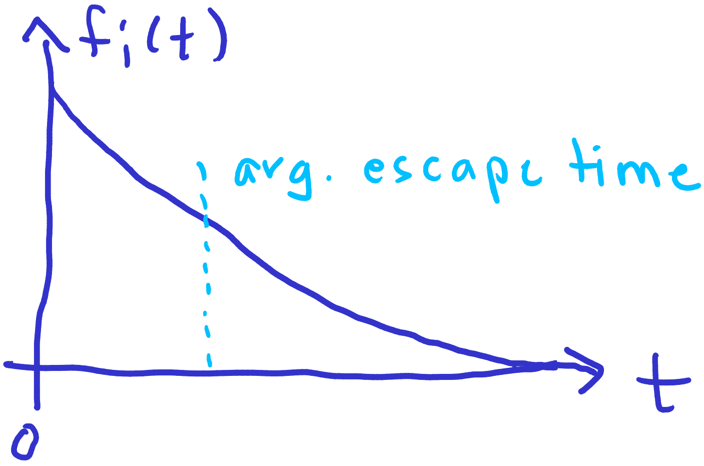
$$\delta t\, f_i(t) = \left[1 - |\Omega_{ii}|\,\delta t\right]^n |\Omega_{ii}|\,\delta t$$

$$\delta t = \frac{t}{n}$$

$$\Rightarrow \quad \delta t\, f_i(t) = \left(1 - |\Omega_{ii}|\frac{t}{n}\right)^n |\Omega_{ii}|\,\delta t$$

small $\delta t \Rightarrow$ large n

$$\Rightarrow \quad f_i(t) = \exp(-|\Omega_{ii}|\, t)\, |\Omega_{ii}|$$



avg. escape time $\tau_i^{esc} = \int_0^\infty dt \; t \, f_i(t)$

$$= \frac{1}{|\Omega_{ii}|}$$

i.e $|\Omega_{ii}| = 3\, s^{-1}$

$$\tau_i^{esc} = \frac{1}{3}\, s$$

2) After escaping, which state did we end up in?

frog problem: prob. of dying via truck

$$= \frac{w_1 \,\delta t}{w_1 \,\delta t + w_2 \,\delta t}$$

$$= \frac{w_1}{w_1 + w_2}$$

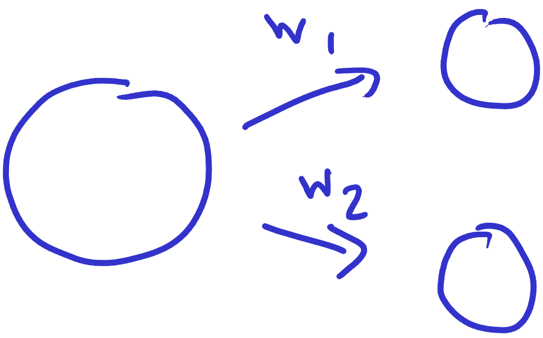
$$\text{Prob. of meteor death} = \frac{w_2}{w_1 + w_2}$$

in general $$\pi_{ji} = \frac{\Omega_{ji}}{|\Omega_{ii}|}$$

π_{ji} ↓ prob. of going to j after leaving i

$$\text{frog alive} \xrightarrow{w_2} \text{meteor death}$$

$$\tau^{esc} = \frac{1}{w_2} = 10^{20}\ \text{s}$$



$$\tau^{esc} = \frac{1}{w_1 + w_2}$$

≈ 20 s

regardless of cause of death

note: probabilities of transition Ω_{ji} are time-independent