PHYS 320/420 Lecture 11

$$k_{smol} = 4\pi D R c$$

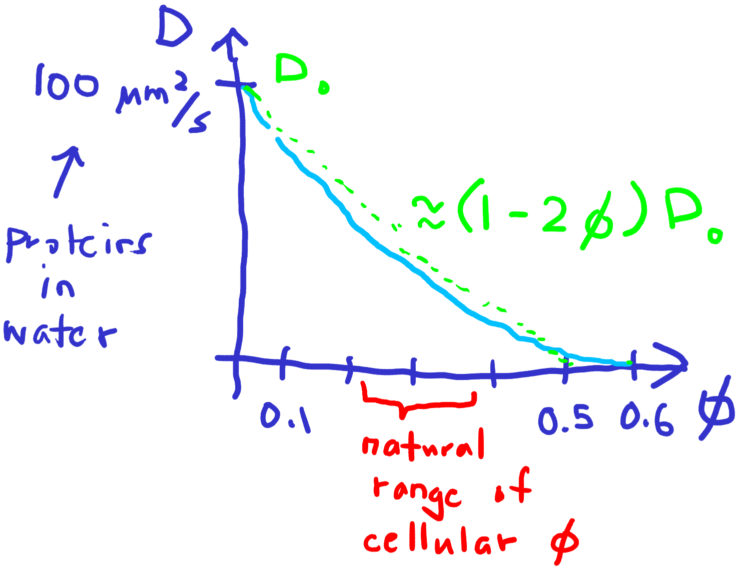
$D \to D_1 + D_2$, $R \to R_1 + R_2$, c: conc $= \frac{n}{V}$

Problem: if we want biological reactions to occur quickly
$\Rightarrow$ crank up the c

but as c increases, cell gets more crowded $\Rightarrow$ D gets smaller

ϕ = volume fraction of stuff in cell: proteins, DNA/RNA, etc.

D for typical proteins (1 nm sized objects)



tradeoff:
inc. ϕ $\Rightarrow$ dec. D

Rough estimate: one type of searcher, one target

$$k = 4\pi D R c$$

$n = \#$ searchers

$$D \cong D_0 (1 - 2\phi)$$

$V_{prot} =$ typ. protein volume = volume of one searcher $\approx 60\ nm^3$

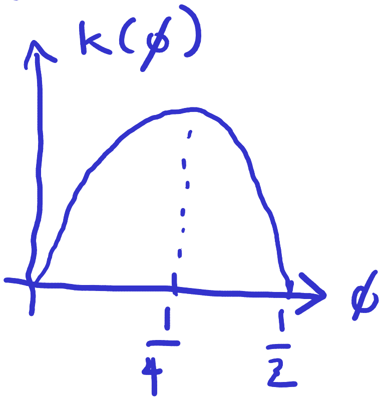
$$\phi \cong \frac{n V_{prot}}{V}$$

V → tot. volume of cell

$$= c V_{prot}$$

$$c = \frac{\phi}{V_{prot}} \Rightarrow k(\phi) \quad \text{maximize this!}$$

$$k(\phi) = 4\pi D_0 (1-2\phi) R \phi V_{prot}^{-1}$$



$$\frac{dk}{d\phi} = 0 \Rightarrow \phi = \frac{1}{4}$$

sweet spot for max. k

$$c = \frac{\phi}{V_{prot}} = 7\ mM$$

$M = \frac{N_A}{L}$ unit of concentration

$V \sim 1\ \mu m^3$ for bacteria

$n = cV = 4 \times 10^6$ proteins in a bacterial cell

E. Coli: $3\times10^6 - 10^7$ proteins dep. on growth rate

optimal $k \simeq 2.6\times10^6\ s^{-1}$

$\tau = 0.4\ \mu s$

Broad lessons (noting there are actually many diff. protein types doing many diff. reactions):

for each type, $c < mM$

$n < 10^6$ for $V = 1\,\mu m^3$

E. Coli: for indiv. proteins

$c \approx 10\ nM - 0.1\ mM$

$n \approx 10 - 10^5$ per cell

Keep in mind that really DNA/RNA also need to be factored in:

$$\phi \approx \frac{n_{prot} V_{prot} + n_{bp} V_{bp}}{V}$$

$n_{bp} \simeq$ # of base pairs of DNA/RNA

$V_{bp} \approx 1\ nm^3$

E. Coli: $\frac{n_{bp} v_{bp}}{V} \approx 0.005$